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1988 J. Phys. A: Math. Gen. 21 699

(http://iopscience.iop.org/0305-4470/21/3/025)

J. Phys. A: Math. Gen. 21 (1988) 699-707. Printed in the UK

Photon correlation effects in scattering

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Received 14 July 1987, in final form 24 September 1987

Abstract. A new theoretical treatment of scattering in the presence of a low-frequency radiation field is presented. A modification of the Kroll-Watson formula, caused by the stochastic properties of the radiation field, is obtained. It is shown that the radiationless cross section remains on the mass shell, but an appropriate shift of the initial and final momenta strongly depends on the stochastic model used. An analytical calculation is carried out for the chaotic field model.

1. Introduction

During the last few years much effort has been invested in the study of scattering in the presence of strong radiation fields (Mittleman 1982, Rosenberg 1982a, Faisal 1984, Gavrila 1985). The basic problems in this context consist in studying the plasma heating by lasers and the laser-driven fusion or in understanding the working principles of gas lasers (especially high-power) and their development. There are also astrophysical motivations consisting in studying plasmas that make up the core of stars, where a high density of radiant energy has to be taken into account. Accurate scattering calculations are, however, difficult even in the absence of the radiation field. A non-perturbative inclusion of the latter would make this problem prohibitively difficult. This is why, except for some general formal approaches, only limiting cases and/or tractable models have been dealt with. The analysis of limiting cases not only greatly simplifies calculations but also more physical insights into the problem can be gained. In this paper the low-frequency approximation of Kroll and Watson (1973) is considered.

It is obvious that no direct comparison between theory and experiment can be achieved without taking into account the realistic behaviour of radiation fields. This requires, on the experimental side, a better knowledge of the characteristics of radiation fields, and on the theoretical side, the development of appropriate models for treating more realistically the influence of light on quantum processes. It is known that light sources (like lasers) produce light which is not exactly monochromatic. The final width of the light signal comes from a smooth time dependence of the field's envelope and/or from rapid chaotic fluctuations of the field's amplitude and phase. The dynamics of scattering in the presence of radiation fields depends on these fluctuations, which are described by stochastic mathematical models. The most commonly used are the phase

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diffusion model, the chaotic field model and the Gaussian-amplitude field model. Descriptions of these models can be found in the review article by Daniele *et al* (1985).

Photon correlation effects in scattering seem to have been considered for the first time by Zoller (1980). In this and all subsequent papers (Trombetta *et al* 1985, Daniele *et al* 1983, Franken and Joachain 1987) only the Born approximation was considered. The Born approximation was also applied to the electron-hydrogen scattering in a chaotic field (Unnikrishnan and Prasad 1986). In this paper I report the generalisation of the low-frequency Kroll-Watson formula, which takes into account stochastic properties of radiation fields. It is shown that apart from the usual modification of the radiation-dependent factor, which multiplies the elastic scattering cross section of the radiationless process, the shift of momenta is also modified. It appears, however, that this modification does not bring the radiationless scattering amplitude out of the mass shell.

2. Scattering matrix in low-frequency radiation fields

Let me, then, consider the non-relativistic scattering of a charged particle by a local and short range potential $V(\boldsymbol{r})$ in the presence of a radiation field. The light field is assumed to be treated classically as a plane wave in the dipole approximation. The electromagnetic vector potential $\boldsymbol{A}(t)$ in this approximation adopts the following form (using units in which $\hbar = 1 = c$):

$$\boldsymbol{A}(t) = \tfrac{1}{2}\mathrm{i}\int_0^\infty \mathrm{d}\Omega\, (\boldsymbol{a}^*(\Omega)\,\mathrm{e}^{\mathrm{i}\Omega t} - \boldsymbol{a}(\Omega)\,\mathrm{e}^{-\mathrm{i}\Omega t}) \tag{1}$$

where $\boldsymbol{a}(\Omega)$ is a complex function. I shall further assume that the spectrum of frequencies Ω is effectively bounded by a value Ω_0, which is much smaller than the projectile's energy, i.e. I shall adopt here the conditions of Kroll and Watson (1973).

The S-matrix element S_{fi} for a transition $\boldsymbol{p}_\mathrm{i} \to \boldsymbol{p}_\mathrm{f}$ can be written as follows:

$$S_{\mathrm{fi}} = \delta(\boldsymbol{p}_\mathrm{i} - \boldsymbol{p}_\mathrm{f}) - \mathrm{i}(2\pi)^{-3}\int \mathrm{d}t\,\mathrm{d}^3r\,\psi_{\boldsymbol{p}_\mathrm{f}}^{(0)*}(\boldsymbol{r}, t)\,V(\boldsymbol{r})\psi_{\boldsymbol{p}_\mathrm{i}}^{(+)}(\boldsymbol{r}, t) \tag{2}$$

where

$$\psi_{\boldsymbol{p}_\mathrm{f}}^{(0)}(\boldsymbol{r}, t) = \exp(-\mathrm{i}E_\mathrm{f}t + i\boldsymbol{p}_\mathrm{f}\cdot\boldsymbol{r} - \mathrm{i}\boldsymbol{p}_\mathrm{f}\cdot\boldsymbol{\alpha}(t)) \tag{3}$$

is the Volkov solution, and

$$\boldsymbol{\alpha}(t) = -\frac{e}{2m}\int_0^\infty \mathrm{d}\Omega\,\Omega^{-1}(\boldsymbol{a}^*(\Omega)\,\mathrm{e}^{\mathrm{i}\Omega t} + \boldsymbol{a}(\Omega)\,\mathrm{e}^{-\mathrm{i}\Omega t}). \tag{4}$$

In (2) the wavefunction $\psi_{\boldsymbol{p}_\mathrm{i}}^{(+)}$ fulfils the Schrödinger equation

$$\mathrm{i}\partial_t\psi_{\boldsymbol{p}_\mathrm{i}}^{(+)} = \left(H_0 + \frac{\mathrm{i}e}{m}\boldsymbol{A}(t)\cdot\boldsymbol{\nabla}\right)\psi_{\boldsymbol{p}_\mathrm{i}}^{(+)} \tag{5}$$

with

$$H_0 = -\frac{1}{2m}\Delta + V(\boldsymbol{r}). \tag{6}$$

In order to find an approximate solution of (5), corrections to which are of the order of Ω_0^2/E_i^2 (E_i is the projectile's kinetic energy, $E_i = p_i^2/2m$), we make the following substitution:

$$\psi_{p_i}^{(+)}(r, t) = \exp(-iE_i t - im\dot{\alpha}(t)\cdot r - ip_i\cdot\alpha(t))\phi_{p_i}(r, t) \tag{7}$$

where $\dot{\alpha}(t)$ means the time derivative of $\alpha(t)$. It can easily be checked that ϕ_{p_i} satisfies the equation

$$(E_i(t) - H_0)\phi_{p_i}(r, t) = -(i\partial_t + m\ddot{\alpha}(t)\cdot r)\phi_{p_i}(r, t) \tag{8}$$

in which

$$E_i(t) = p_i^2(t)/2m \qquad p_i(t) = p_i + m\dot{\alpha}(t). \tag{9}$$

The solution of (8) (satisfying the scattering boundary conditions) fulfils the integral equation

$$\phi_{p_i}(r, t) = \psi_{p_i(t)}^{(+)}(r) - \int d^3r' G^{(+)}(r, r'; E_i(t))(i\partial_t + m\ddot{\alpha}\cdot r')\phi_{p_i}(r', t) \tag{10}$$

which can be solved iteratively. In the above equation $\psi_{p_i(t)}^{(+)}$ is the stationary scattering wavefunction with momentum $p_i(t)$ (9) modified by the radiation field and $G^{(+)}$ is the Green function

$$(E - H_0)G^{(+)}(r, r'; E) = \delta(r - r') \tag{11}$$

satisfying the boundary conditions with outgoing spherical waves. We can prove now that the contribution to the scattering matrix (2) of the last term in (10) is if the order of Ω_0^2/E_i^2. Indeed, the second derivative of $\alpha(t)$ behaves effectively like $\Omega_0^2\alpha(t)$. Moreover, the wavefunction $\psi_{p_i(t)}^{(+)}$ fulfils the integral equation

$$\psi_{p_i(t)}^{(+)}(r) = \psi_{p_i}^{(+)}(r) - (m\dot{\alpha}(t)\cdot p_i + m\dot{\alpha}^2(t)/2)\int d^3r'\, G^{(+)}(r, r'; E_i)\psi_{p_i(t)}^{(+)}(r')$$

which again can be solved iteratively. It follows from such a formal solution that the time derivative of $\psi_{p_i(t)}^{(+)}$ is proportional to $\ddot{\alpha}(t) \approx \Omega_0^2\alpha(t)$. This means that the solution of (5) can be written in the form

$$\psi_{p_i}^{(+)}(r, t) - \exp(-iE_i t - im\dot{\alpha}(t)\cdot r - i\alpha(t)\cdot p_i)\psi_{p_i(t)}^{(+)}(r) \tag{12}$$

plus terms whose contribution to the scattering matrix (2) is of the order of Ω_0^2/E_i^2. Substituting the above approximation into the exact expression for the scattering matrix we find that the S-matrix element for a transition $p_i \to p_f$, exact up to terms quadratic in Ω_0/E_i, can be written as follows:

$$S_{fi} = \delta(p_i - p_f) - \frac{i}{4\pi^2 m}\int dt\, \exp(iE_{fi}t + i\Delta_{fi}\cdot\alpha(t) + m\dot{\alpha}(t)\cdot\nabla_{fi})f(p_i, p_f) + O(\Omega_0^2/E_i^2) \tag{13}$$

where

$$f(p_i, p_f) = -\frac{m}{2\pi}\int d^3r\, \exp(-ip_f\cdot r)V(r)\psi_{p_i}^{(+)}(r) \tag{14}$$

is the off-shell radiationless scattering amplitude. Moreover, in (13), $E_{\mathrm{fi}}=E_{\mathrm{f}}-E_{\mathrm{i}}$, $\boldsymbol{\Delta}_{\mathrm{fi}}=\boldsymbol{p}_{\mathrm{f}}-\boldsymbol{p}_{\mathrm{i}}$ and $\boldsymbol{\nabla}_{\mathrm{fi}}=\partial/\partial\boldsymbol{p}_{\mathrm{f}}+\partial/\partial\boldsymbol{p}_{\mathrm{i}}$. Let us note in passing that for $\boldsymbol{\alpha}(t)=\boldsymbol{\alpha}_0\sin(\omega t)$ we obtain the Kroll–Watson formula,

$$S_{\mathrm{fi}}=\delta(\boldsymbol{p}_{\mathrm{i}}-\boldsymbol{p}_{\mathrm{f}})+\frac{\mathrm{i}}{2\pi m}\sum_{n=-\infty}^{\infty}\delta(E_{\mathrm{fi}}-n\omega)J_n(-\boldsymbol{\alpha}_0\cdot\boldsymbol{\Delta}_{\mathrm{fi}})f(\boldsymbol{p}_{\mathrm{i}}-\boldsymbol{\lambda}_n,\boldsymbol{p}_{\mathrm{f}}-\boldsymbol{\lambda}_n)+\mathrm{O}(\omega^2/E_{\mathrm{i}}^2) \tag{15}$$

where

$$\boldsymbol{\lambda}_n=n\omega m\boldsymbol{\alpha}_0/(\boldsymbol{\alpha}_0\cdot\boldsymbol{\Delta}_{\mathrm{fi}}). \tag{16}$$

Moreover, when $f(\boldsymbol{p}_{\mathrm{i}},\boldsymbol{p}_{\mathrm{f}})$ in (13) becomes proportional to the Fourier transform of the potential, $V(\boldsymbol{\Delta}_{\mathrm{fi}})$, we get the well known expression for the scattering matrix in the Born approximation.

With the help of expression (13) one obtains for the non-forward collision process the following equation:

$$|S_{\mathrm{fi}}|^2=\frac{1}{(2\pi)^4m^2}\int \mathrm{d}t\,\mathrm{d}t'\exp(\mathrm{i}E_{\mathrm{fi}}t-\mathrm{i}E'_{\mathrm{fi}}t')$$
$$\times I_{\mathrm{fi}}(t,t')f(\boldsymbol{p}_{\mathrm{i}},\boldsymbol{p}_{\mathrm{f}})f^*(\boldsymbol{p}'_{\mathrm{i}},\boldsymbol{p}'_{\mathrm{f}})|_{\boldsymbol{p}'_{\mathrm{i}}=\boldsymbol{p}_{\mathrm{i}},\boldsymbol{p}'_{\mathrm{f}}=\boldsymbol{p}_{\mathrm{f}}}+\mathrm{O}(\Omega_0^2/E_{\mathrm{i}}^2) \tag{17}$$

where

$$I_{\mathrm{fi}}(t,t')=\exp(\mathrm{i}\boldsymbol{\Delta}_{\mathrm{fi}}\cdot\boldsymbol{\alpha}(t)-\mathrm{i}\boldsymbol{\Delta}'_{\mathrm{fi}}\cdot\boldsymbol{\alpha}(t')+m\dot{\boldsymbol{\alpha}}(t)\cdot\boldsymbol{\nabla}_{\mathrm{fi}}+m\dot{\boldsymbol{\alpha}}(t')\cdot\boldsymbol{\nabla}'_{\mathrm{fi}}) \tag{18}$$

and E'_{fi}, $\boldsymbol{\Delta}'_{\mathrm{fi}}$ and $\boldsymbol{\nabla}'_{\mathrm{fi}}$ are defined with p'_{i} and p'_{f} (i.e. $\boldsymbol{\nabla}'_{\mathrm{fi}}=\partial/\partial\boldsymbol{p}'_{\mathrm{f}}+\partial/\partial\boldsymbol{p}'_{\mathrm{i}}$, etc).

3. Chaotic field model

Let us consider initially the simplest case of the chaotic field model with the vanishing bandwidth. The last simplification is not very restrictive because, as has been shown (Trombetta *et al* 1985, Daniele *et al* 1983, Franken and Joachain 1987) the only effect of the non-zero bandwidth is to give a spread of δ-like peaks in the double differential cross section $\mathrm{d}^2\sigma/\mathrm{d}\Omega\,\mathrm{d}E_{\mathrm{f}}$ at final energies corresponding to the exchange of an integral number of photons.

In order to determine the ensemble average of $|S_{\mathrm{fi}}|^2$, it is sufficient to calculate such an average for the function $I_{\mathrm{fi}}(t,t')$. It is known (Zoller 1980, Daniele *et al* 1985, Gardiner 1983) that for the chaotic field model the function $\boldsymbol{\alpha}(t)$ is a Gaussian stochastic process, and that

$$\langle I_{\mathrm{fi}}(t,t')\rangle=\exp[-\tfrac{1}{2}\langle(\boldsymbol{\Delta}_{\mathrm{fi}}\cdot\boldsymbol{\alpha}(t)-\boldsymbol{\Delta}'_{\mathrm{fi}}\cdot\boldsymbol{\alpha}(t')-\mathrm{i}m\dot{\boldsymbol{\alpha}}(t)\boldsymbol{\nabla}_{\mathrm{fi}}-\mathrm{i}m\dot{\boldsymbol{\alpha}}(t')\boldsymbol{\nabla}'_{\mathrm{fi}})^2\rangle] \tag{19}$$

where the angular brackets denote ensemble averaging. The ensemble average in the exponent of (19) for the nearly monochromatic radiation field can be calculated by writing the vector $\boldsymbol{\alpha}(t)$ in the form

$$\boldsymbol{\alpha}(t)=\boldsymbol{\mathscr{A}}(t)\,\mathrm{e}^{-\mathrm{i}\omega t}+\boldsymbol{\mathscr{A}}^*(t)\,\mathrm{e}^{\mathrm{i}\omega t} \tag{20}$$

where $\boldsymbol{\mathscr{A}}(t)$ is a stochastic amplitude of $\boldsymbol{\alpha}(t)$ slowly varying in time. The correlation functions of the complex amplitude $\boldsymbol{\mathscr{A}}(t)$ are assumed to be constants (for the vanishing bandwidth). For the unpolarised radiation field we have

$$\langle\mathscr{A}_i(t)\mathscr{A}_j^*(t')\rangle=\tfrac{1}{2}D\delta_{ij}\qquad\langle\mathscr{A}_i(t)\mathscr{A}_j(t')\rangle=0. \tag{21}$$

With these correlation functions we arrive, after some simple manipulation, at

$$\langle(\boldsymbol{\Delta}_{\mathrm{fi}}\cdot\boldsymbol{\alpha}(t)-\boldsymbol{\Delta}'_{\mathrm{fi}}\cdot\boldsymbol{\alpha}(t')-\mathrm{i}m\dot{\boldsymbol{\alpha}}(t)\cdot\boldsymbol{\nabla}_{\mathrm{fi}}-\mathrm{i}m\dot{\boldsymbol{\alpha}}(t')\cdot\boldsymbol{\nabla}'_{\mathrm{fi}})^2\rangle$$
$$=D(\boldsymbol{\Delta}^2_{\mathrm{fi}}+\boldsymbol{\Delta}'^2_{\mathrm{fi}})-2D\boldsymbol{\Delta}_{\mathrm{fi}}\cdot\boldsymbol{\Delta}'_{\mathrm{fi}}\cos[\omega(t-t')]$$
$$+2\mathrm{i}m\omega D(\boldsymbol{\Delta}_{\mathrm{fi}}\cdot\boldsymbol{\nabla}'_{\mathrm{fi}}+\boldsymbol{\Delta}'_{\mathrm{fi}}\cdot\boldsymbol{\nabla}_{\mathrm{fi}})\sin[\omega(t-t')] \quad (22)$$

plus terms proportional to ω^2, thus leading to corrections to the scattering matrix which are of the order of $\omega^2/E_{\mathrm{i}}^2$, and where the time derivative of the complex amplitude $\mathscr{A}(t)$ has been neglected. Taking account of the well known properties of the Bessel functions with imaginary argument, i.e.

$$\mathrm{e}^{z\cos\omega t}=\sum_{n=-\infty}^{\infty}\mathrm{e}^{-\mathrm{i}n\omega t}I_n(z) \quad (23)$$

$$I_{n+1}(z)-I_{n-1}(z)=-(2n/z)I_n(z) \quad (24)$$

one immediately obtains

$$\langle I_{\mathrm{fi}}(t,t')\rangle=\sum_{n=-\infty}^{\infty}\exp[-\mathrm{i}n\omega(t-t')]\exp[-\tfrac{1}{2}D(\boldsymbol{\Delta}^2_{\mathrm{fi}}+\boldsymbol{\Delta}'^2_{\mathrm{fi}})]$$
$$\times I_n(D\boldsymbol{\Delta}_{\mathrm{fi}}\cdot\boldsymbol{\Delta}'_{\mathrm{fi}})\exp[-n\omega m(\boldsymbol{\Delta}_{\mathrm{fi}}\cdot\boldsymbol{\nabla}'_{\mathrm{fi}}+\boldsymbol{\Delta}'_{\mathrm{fi}}\cdot\boldsymbol{\nabla}_{\mathrm{fi}})/\boldsymbol{\Delta}_{\mathrm{fi}}\cdot\boldsymbol{\Delta}'_{\mathrm{fi}}] \quad (25)$$

plus terms proportional to ω^2.

With the help of this result it is straightforward now to obtain the transition probability w_{fi} per unit time,

$$w_{\mathrm{fi}}=\frac{1}{8\pi^3m^2}\sum_{n=-\infty}^{\infty}\delta(E_{\mathrm{fi}}-n\omega)\exp(-D\boldsymbol{\Delta}^2_{\mathrm{fi}})I_n(D\boldsymbol{\Delta}^2_{\mathrm{fi}})$$
$$\times|f(\boldsymbol{p}_{\mathrm{i}}-\boldsymbol{\lambda}_n^{\mathrm{CH}},\boldsymbol{p}_{\mathrm{f}}-\boldsymbol{\lambda}_n^{\mathrm{CH}})|^2+\mathrm{O}(\omega^2/E_{\mathrm{i}}^2) \quad (26)$$

where the shift $\boldsymbol{\lambda}_n^{\mathrm{CH}}$ is equal to

$$\boldsymbol{\lambda}_n^{\mathrm{CH}}=n\omega m\boldsymbol{\Delta}_{\mathrm{fi}}/\boldsymbol{\Delta}^2_{\mathrm{fi}} \quad (27)$$

and becomes now (i.e. for the chaotic field model) proportional to the momentum transfer $\boldsymbol{\Delta}_{\mathrm{fi}}$. One can easily check that

$$(\boldsymbol{p}_{\mathrm{i}}-\boldsymbol{\lambda}_n^{\mathrm{CH}})^2=(\boldsymbol{p}_{\mathrm{f}}-\boldsymbol{\lambda}_n^{\mathrm{CH}})^2 \quad (28)$$

i.e. all radiationless scattering amplitudes in (26) remain on the mass shell. We will see that this property holds in general, independently of the stochastic models used.

Equation (26) allows us to put down the differential cross section for the scattering process with n photons absorbed ($n>0$) or emitted ($n<0$) as follows:

$$\frac{\mathrm{d}\sigma_n^{\mathrm{CH}}}{\mathrm{d}\Omega}=\frac{p_{\mathrm{f}}}{p_{\mathrm{i}}}\exp(-D\boldsymbol{\Delta}^2_{\mathrm{fi}})I_n(D\boldsymbol{\Delta}^2_{\mathrm{fi}})|f(\boldsymbol{p}_i-\boldsymbol{\lambda}_n^{\mathrm{CH}},\boldsymbol{p}_{\mathrm{f}}-\boldsymbol{\lambda}_n^{\mathrm{CH}})|^2 \quad (29)$$

where, due to the conservation of energy, $\boldsymbol{p}_{\mathrm{f}}^2=\boldsymbol{p}_{\mathrm{i}}^2+2mn\omega$. This is the required generalisation of the Kroll–Watson formula (Kroll and Watson 1973) and of the result presented by Zoller (1980) to the case of the chaotic field model. Using (13) for the scattering matrix in the presence of low-frequency radiation fields, one can immediately (at least formally) obtain such a generalisation to an arbitrary stochastic model.

Let us also consider another example of the chaotic field model in which

$$\boldsymbol{\alpha}(t)=\boldsymbol{\alpha}_0\sin(\omega t)+\boldsymbol{\alpha}_{\mathrm{S}}(t) \quad (30)$$

where

$$\boldsymbol{\alpha}_S(t) = \boldsymbol{\alpha}_G(t)\,e^{-i\omega t} + \boldsymbol{\alpha}_G^*(t)\,e^{i\omega t} \tag{31}$$

with a complex amplitude $\boldsymbol{\alpha}_G(t)$ slowly varying in time. It is assumed that $\boldsymbol{\alpha}_G(t)$ has Gaussian stochastic properties (with the zero bandwidth), i.e.

$$\langle \alpha_{Gi}(t)\alpha_{Gj}^*(t')\rangle = \tfrac{1}{2}D_{ij} \qquad \langle \alpha_{Gi}(t)\alpha_{Gj}(t')\rangle = 0 \tag{32}$$

where $(\hat{D})_{ij} = D_{ij}$ is a constant matrix. After some simple algebraic manipulations we arrive at the following expression for the cross section:

$$\frac{d\sigma_n^S}{d\Omega} = \frac{p_f}{p_i}\tilde{I}_n(\boldsymbol{\Delta}_{fi}, \boldsymbol{\alpha}_0)|f(\boldsymbol{p}_i - \boldsymbol{\lambda}_n^S, \boldsymbol{p}_f - \boldsymbol{\lambda}_n^S)|^2 \tag{33}$$

where

$$\tilde{I}_n(\boldsymbol{\Delta}_{fi}, \boldsymbol{\alpha}_0) = \exp(-\boldsymbol{\Delta}_{fi}\hat{D}\boldsymbol{\Delta}_{fi}) \sum_{k=-\infty}^{\infty} J_{n-k}^2(\boldsymbol{\alpha}_0 \cdot \boldsymbol{\Delta}_{fi}) I_k(\boldsymbol{\Delta}_{fi}\hat{D}\boldsymbol{\Delta}_{fi}) \tag{34}$$

and $\boldsymbol{\lambda}_n^S$ can be determined from the equation

$$\boldsymbol{\lambda}_n^S \tilde{I}_n(\boldsymbol{\Delta}_{fi}, \boldsymbol{\alpha}_0) = m\omega \exp(-\boldsymbol{\Delta}_{fi}\hat{D}\boldsymbol{\Delta}_{fi}) \sum_{k=-\infty}^{\infty} J_{n-k}^2(\alpha_0 \cdot \boldsymbol{\Delta}_{fi}) I_k(\boldsymbol{\Delta}_{fi}\hat{D}\boldsymbol{\Delta}_{fi}) \times \left(\frac{(n-k)\boldsymbol{\alpha}_0}{\boldsymbol{\alpha}_0\boldsymbol{\Delta}_{fi}} + \frac{k\boldsymbol{\Delta}_{fi}\hat{D}}{\boldsymbol{\Delta}_{fi}\hat{D}\boldsymbol{\Delta}_{fi}}\right). \tag{35}$$

It can be checked that the radiationless scattering amplitude remains on the mass shell. Moreover, with $\hat{D} = 0$ we recover the Kroll–Watson formula, and with $\boldsymbol{\alpha}_0 = 0$ and $D_{ij} = D\delta_{ij}$ we obtain (29).

4. Arbitrary stochastic field model

Having illustrated the application of (13) to the simple chaotic model let us show now that the results obtained previously can be generalised to the case of an arbitrary stochastic field model. To this end let us note that

$$\langle I_{fi}(t,t')\rangle = I(\boldsymbol{\Delta}_{fi}, t; \boldsymbol{\Delta}'_{fi}, t') + \boldsymbol{J}(\boldsymbol{\Delta}_{fi}, t; \boldsymbol{\Delta}'_{fi}, t') \cdot \boldsymbol{\nabla}_{fi} + \boldsymbol{J}^*(\boldsymbol{\Delta}'_{fi}, t'; \boldsymbol{\Delta}_{fi}, t) \cdot \boldsymbol{\nabla}'_{fi} + O(\Omega_0^2/E_i^2) \tag{36}$$

where

$$I(\boldsymbol{\Delta}_{fi}, t; \boldsymbol{\Delta}'_{fi}, t') = I^*(\boldsymbol{\Delta}'_{fi}, t'; \boldsymbol{\Delta}_{fi}, t) = \langle \exp(i\boldsymbol{\Delta}_{fi} \cdot \boldsymbol{\alpha}(t) - i\boldsymbol{\Delta}'_{fi} \cdot \boldsymbol{\alpha}(t'))\rangle \tag{37}$$

and

$$\boldsymbol{J}(\boldsymbol{\Delta}_{fi}, t; \boldsymbol{\Delta}'_{fi}, t') = m\langle \dot{\boldsymbol{\alpha}}(t) \exp(i\boldsymbol{\Delta}_{fi} \cdot \boldsymbol{\alpha}(t) - i\boldsymbol{\Delta}'_{fi} \cdot \boldsymbol{\alpha}(t'))\rangle. \tag{38}$$

Defining new functions $\tilde{I}(\boldsymbol{\Delta}_{fi}, \Omega; \boldsymbol{\Delta}'_{fi}, \Omega')$ and $\boldsymbol{j}(\boldsymbol{\Delta}_{fi}, \Omega; \boldsymbol{\Delta}'_{fi}, \Omega')$ through the equations

$$I(\boldsymbol{\Delta}_{fi}, t; \boldsymbol{\Delta}'_{fi}, t') = \int d\Omega\, d\Omega' \exp(-i\Omega t + i\Omega' t')\tilde{I}(\boldsymbol{\Delta}_{fi}, \Omega; \boldsymbol{\Delta}'_{fi}, \Omega') \tag{39}$$

and

$$J(\mathbf{\Delta}_{\mathrm{fi}}, t; \mathbf{\Delta}'_{\mathrm{fi}}, t') = \int \mathrm{d}\Omega\, \mathrm{d}\Omega' \exp(-\mathrm{i}\Omega t + \mathrm{i}\Omega' t') \boldsymbol{j}(\mathbf{\Delta}_{\mathrm{fi}}, \Omega; \mathbf{\Delta}'_{\mathrm{fi}}, \Omega') \tag{40}$$

we arrive, after some simple algebraic manipulations, at the following equation:

$$\langle |S_{\mathrm{fi}}|^2 \rangle = (2\pi m)^{-2} \tilde{I}(\mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}, \mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}) |f(\boldsymbol{p}_{\mathrm{i}} - \boldsymbol{\lambda}(\mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}), \boldsymbol{p}_{\mathrm{f}} - \boldsymbol{\lambda}(\mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}))|^2 \tag{41}$$

in which

$$\boldsymbol{\lambda}(\mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}) = -\boldsymbol{j}(\mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}; \mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}) / \tilde{I}(\mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}; \mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}). \tag{42}$$

One can also prove that the elastic radiationless scattering amplitude in (41) remains on the mass shell. Indeed, it follows from (40) and (38) that

$$\mathbf{\Delta}_{\mathrm{fi}} \cdot \boldsymbol{j}(\mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}, \mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}) = -\mathrm{i}m(2\pi)^2 \int \mathrm{d}t\, \mathrm{d}t' \tag{43}$$

$$\times \exp[\mathrm{i}E_{\mathrm{fi}}(t - t')] \partial_t \langle \exp(\mathrm{i}\mathbf{\Delta}_{\mathrm{fi}} \cdot \boldsymbol{\alpha}(t) - \mathrm{i}\mathbf{\Delta}_{\mathrm{fi}} \cdot \boldsymbol{\alpha}(t')) \rangle.$$

Integrating the last expression by parts we find that

$$\mathbf{\Delta}_{\mathrm{fi}} \cdot \boldsymbol{j}(\mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}; \mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}) = -mE_{\mathrm{fi}} \tilde{I}(\mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}; \mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}). \tag{44}$$

It follows from this equation and from the definition (42) of the momentum shift $\boldsymbol{\lambda}$ that

$$mE_{\mathrm{fi}} = \mathbf{\Delta}_{\mathrm{fi}} \cdot \boldsymbol{\lambda}(\mathbf{\Delta}_{\mathrm{fi}}, E_{\mathrm{fi}}). \tag{45}$$

The last equation expresses indeed the 'on-mass-shell' condition, i.e.

$$(\boldsymbol{p}_{\mathrm{i}} - \boldsymbol{\lambda})^2 = (\boldsymbol{p}_{\mathrm{f}} - \boldsymbol{\lambda})^2. \tag{46}$$

It is to be noted, however, that for the adopted stochastic field model the functions $\tilde{I}$ and $\boldsymbol{\lambda}$ in (41) still remain to be calculated. In this sense this result is formal.

5. Primary conclusions

In this paper a new theoretical treatment of the potential scattering in the presence of a low-frequency radiation field has been presented. It has been shown how terms linear with respect to Ω_0/E_{i} can be accounted for in order to obtain a modified Kroll–Watson formula. The method presented here can be applied to a radiation field of any stochastic properties. I have carried out the analytical calculation for the simplest chaotic field model, showing explicitly how the result obtained by Zoller (1980) is modified by terms proportional to the frequency ω. Without conceptual difficulties one can generalise results obtained for other stochastic models. It appears that in all these cases the radiationless cross section remains on the mass shell. However, the shift of momenta strongly depends on the stochastic model used. It should be noted that such a shift is a universal quantity in the sense that it does not depend on the form of interaction.

6. Perspectives and prospects

How can we improve the approximation (12) of the wavefunction in order also to take account of higher terms of the expansion in Ω_0/E_{i}? The answer to this question, as recently developed by Rosenberg (1982b, 1986), is the so-called modified perturbation theory for scattering in radiation fields. The approximation (12) can be treated as the trial function of the Rosenberg approach. However, in order to develop the systematic iteration procedure also a suitable trial Green function has to be chosen. It can be checked, using the same methods as before, that

$$K_{\mathrm{R}}(\boldsymbol{r}, t; \boldsymbol{r}', t') \approx \mathrm{i}\theta(t-t') \exp(-\mathrm{i}m\dot{\boldsymbol{\alpha}}(t)\cdot\boldsymbol{r}) \int \mathrm{d}^3q \exp[-\mathrm{i}E_{\boldsymbol{q}}(t-t')] \times \psi_{\boldsymbol{q}}^{(+)}(\boldsymbol{r})\psi_{\boldsymbol{q}}^{(+)*}(\boldsymbol{r}') \exp(\mathrm{i}m\dot{\boldsymbol{\alpha}}(t')\cdot\boldsymbol{r}') \tag{47}$$

where the notation $\int \mathrm{d}^3q$ is meant to include a sum over discrete states as well as integration over the continuum, and θ is the step function. In the above equation K_{R} is the retarded Green function which fulfils the equation

$$[\mathrm{i}\partial_t + (\mathrm{i}\nabla + e\boldsymbol{A}(t))^2/2m - V(\boldsymbol{r})]K_{\mathrm{R}}(\boldsymbol{r}, t; \boldsymbol{r}', t') = \delta(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t') \tag{48}$$

and $\psi_{\boldsymbol{q}}^{(+)}$ are the eigenfunctions of H_0 (equation (6)). Treating the approximation (47) as the trial Green function in the Rosenberg approach we can also take account of higher orders of the low-frequency expansion.

The method presented in this paper can be immediately adapted for the scattering of electrons by composed systems (like atom or molecules, but also impurities in crystals). Namely, let $\psi_{\boldsymbol{p}_i}^{(+)}(\boldsymbol{r}_1, \cdots, \boldsymbol{r}_{N+1}, t)$ be the radiationless time-dependent wavefunction that describes the scattering of an electron by an N-electron system. Such a function can be quite well determined by the variational methods (Nesbet 1980). Hence, the following low-frequency approximation holds:

$$\psi_{\boldsymbol{p}_i}^{(+)}(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_{N+1}, t; \boldsymbol{A}) = \exp\left(-\mathrm{i}m \sum_{l=1}^{N+1} \boldsymbol{r}_l\cdot\dot{\boldsymbol{\alpha}}(t) - \mathrm{i}\boldsymbol{p}_i\cdot\boldsymbol{\alpha}(t) - \mathrm{i}\int^t (e^2\boldsymbol{A}^2(t')/2m)\,\mathrm{d}t'\right) \times \psi_{\boldsymbol{p}_i(t)}^{(+)}(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_{N+1}, t) \tag{49}$$

where $\boldsymbol{\alpha}(t)$ and $\boldsymbol{p}_i(t)$ are determined by (4) and (9), respectively, and the wavefunction $\psi_{\boldsymbol{p}_i}^{(+)}(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_{N+1}, t; \boldsymbol{A})$, describing a process in the presence of a radiation field $\boldsymbol{A}(t)$, fulfils the Schrödinger equation

$$\mathrm{i}\partial_t\psi_{\boldsymbol{p}_i}^{(+)} = \left(\frac{1}{2m}\sum_{l=1}^{N+1} (\mathrm{i}\nabla_l + e\boldsymbol{A}(t))^2 + V(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_{N+1})\right)\psi_{\boldsymbol{p}_i}^{(+)}. \tag{50}$$

In the above equation $\nabla_l = \partial/\partial\boldsymbol{r}_l$ and $V(\boldsymbol{r}_1, \cdots, \boldsymbol{r}_{N+1})$ is the static potential describing the interaction of components without the radiation field.

Acknowledgments

The author wishes to thank Professsor F H M Faisal for his warm hospitality at the University of Bielefeld. This work has been supported by the Polish Project for Fundamental Researches (Grant no CPBP 01.03) and by the Deutsche Forschungsgemeinschaft under project SFB-216.

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